Einführung in die Fouriertransformation

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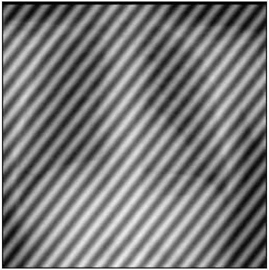

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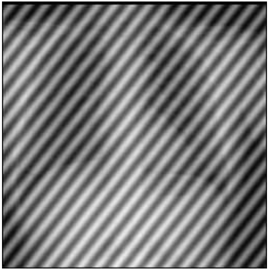

Fouriertrans-
$\longrightarrow$
formation

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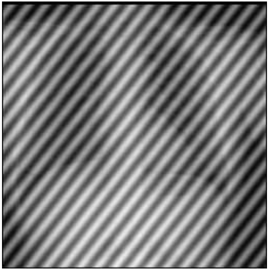
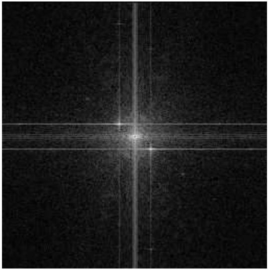


Fouriertrans-
→
formation



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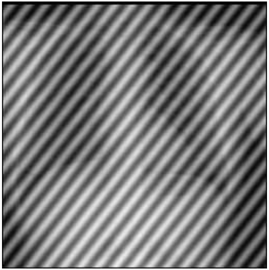

Fouriertrans-

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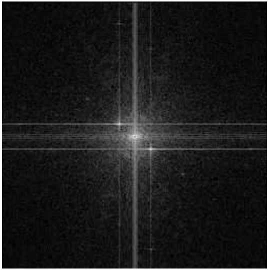

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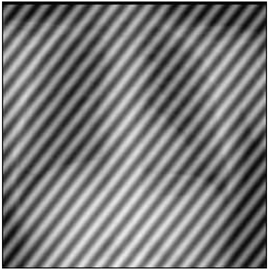

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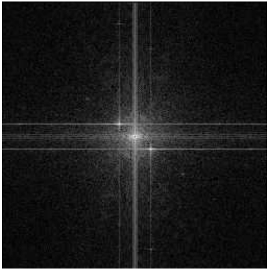

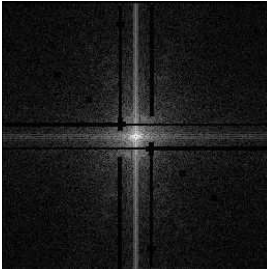

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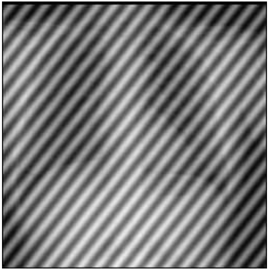

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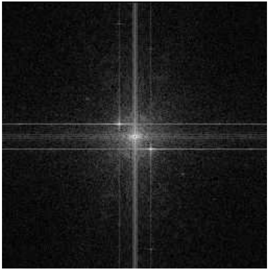
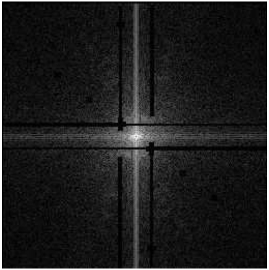
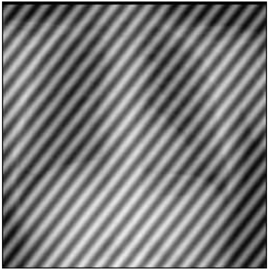


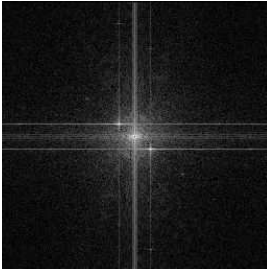


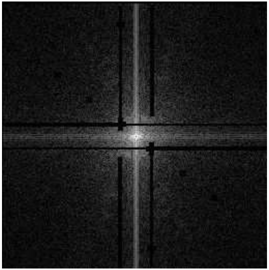
inverse
Fouriertrans-
formation

Einführung in die Fouriertransformation



Fouriertrans-
formation





inverse
Fouriertrans-
formation

Fourier-Entwicklung

Fourier-Entwicklung

Elementare Funktionen:

Fourier-Entwicklung

Elementare Funktionen: 1

Fourier-Entwicklung

Elementare Funktionen: $1, \cos(kx)$

Fourier-Entwicklung

Elementare Funktionen: $1, \cos(kx), \sin(kx)$

Fourier-Entwicklung

Elementare Funktionen: $1, \cos(kx), \sin(kx)$ für $k = 1, 2, 3, \ldots$

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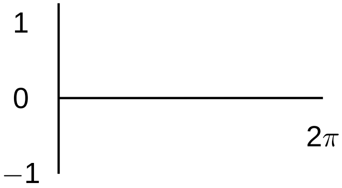

Fourier-Entwicklung

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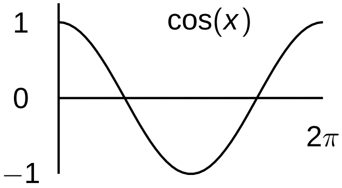

Fourier-Entwicklung

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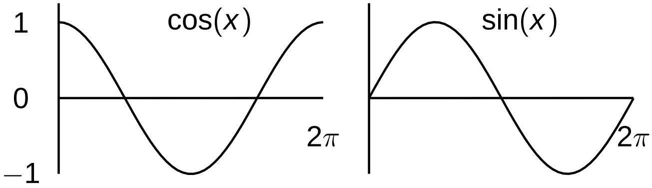

Fourier-Entwicklung

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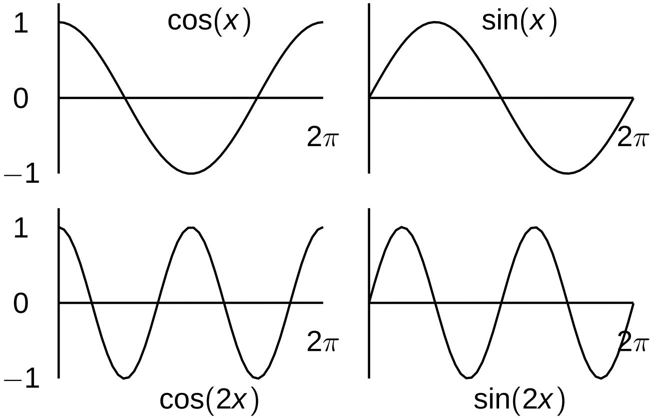

Fourier-Entwicklung

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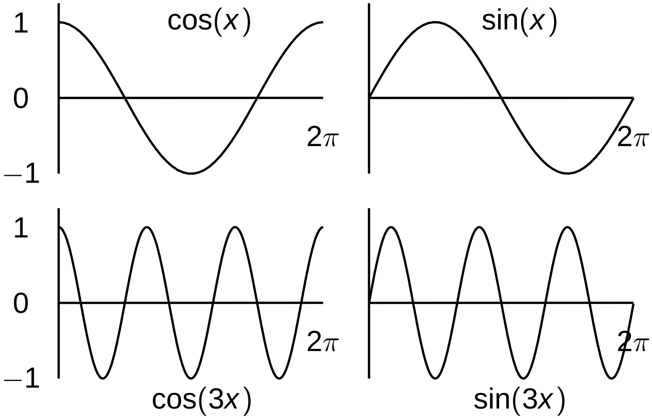

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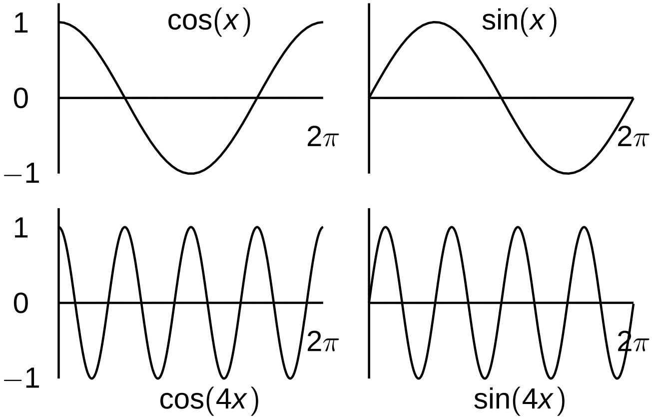

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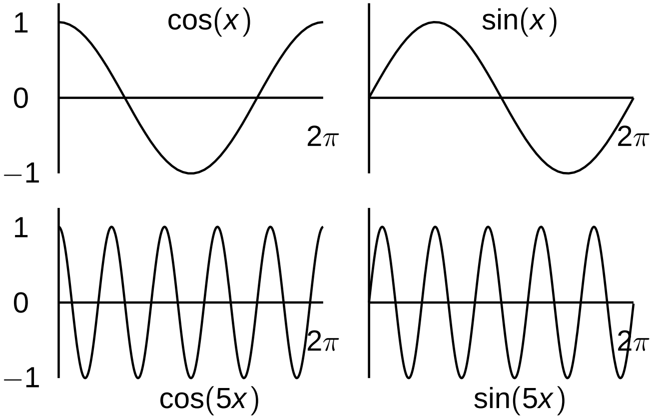

Fourier-Entwicklung

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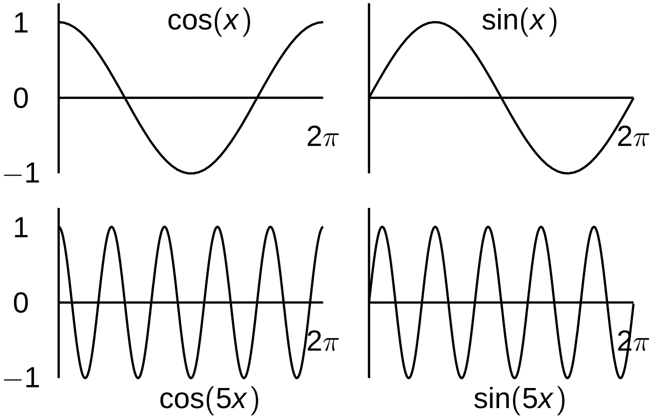

u.s.w.

Fourier-Entwicklung

Fourier-Entwicklung

Beispiel:

$f_1(x) = 0.9 \cdot \cos(x)$

Fourier-Entwicklung

Beispiel:

$f_1(x) = 0.9 \cdot \cos(x)$

$f_2(x) = 0.9 \cdot \cos(x)$

Fourier-Entwicklung

Beispiel:

$f_1(x) = 0.9 \cdot \cos(x)$

$f_2(x) = 0.9 \cdot \cos(x) + 0.5 \cdot \sin(3x)$

Fourier-Entwicklung

Beispiel:

$f_1(x) = 0.9 \cdot \cos(x)$

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Fourier-Entwicklung

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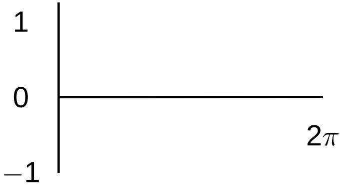

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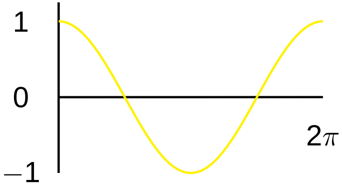

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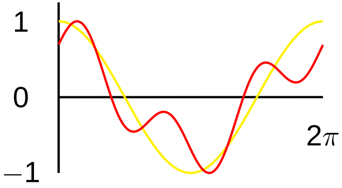

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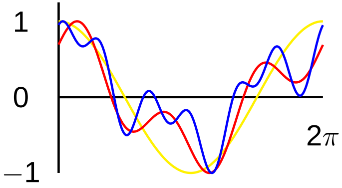

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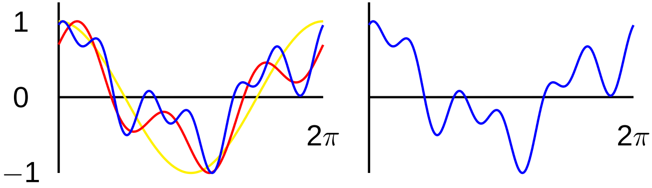

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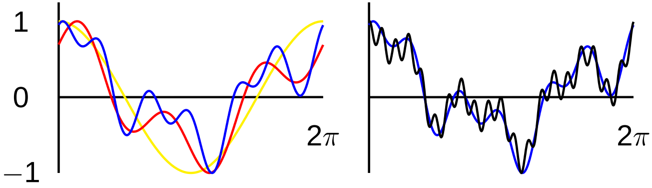

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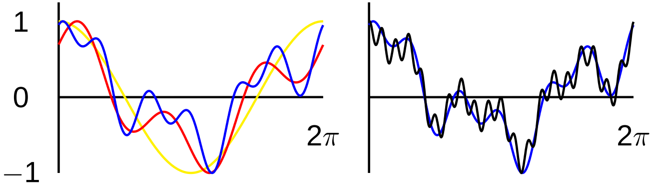

Fourier-Koeffizienten

Fourier-Entwicklung

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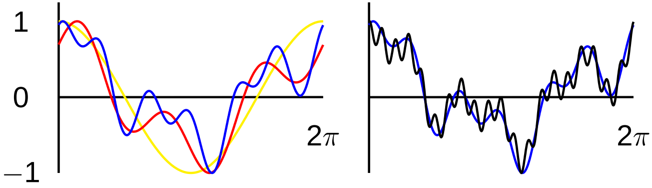

Fourier-Koeffizienten

$f_4(x) = \boxed{0.9} \cdot \cos(x) + \boxed{0.5} \cdot \sin(3x) + \boxed{0.4} \cdot \sin(6x) + \boxed{0.3} \cdot \cos(20x)$

Fourier-Entwicklung

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$

Fourier-Entwicklung

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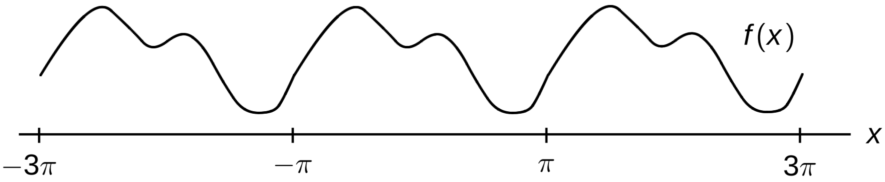
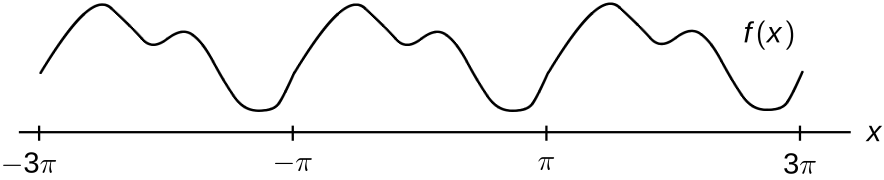


Fourier-Entwicklung

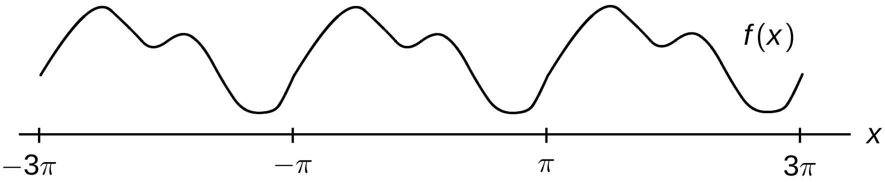
Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Reihe

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$

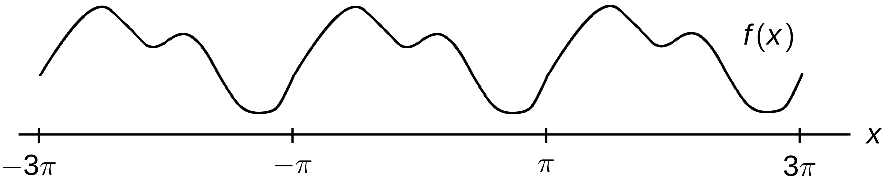


Fourier-Reihe

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$

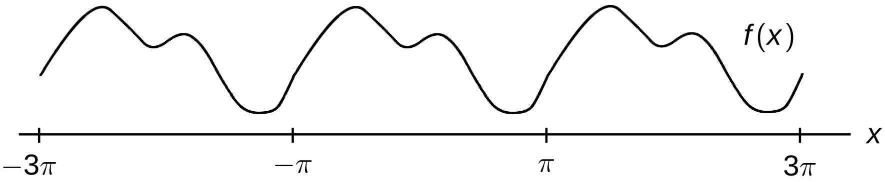


Fourier-Reihe

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx)) = f(x)$$

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Reihe

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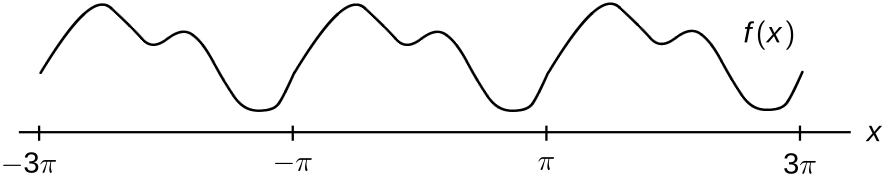
falls man a_k, b_k geschickt wählt, nämlich

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Reihe

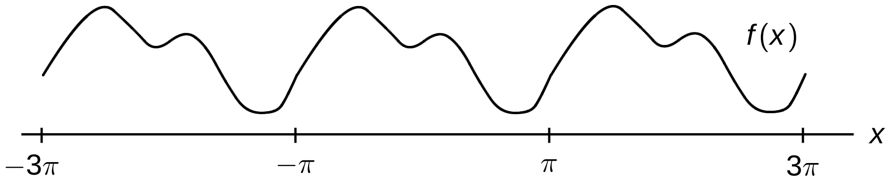
$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx)) = f(x)$$

falls man a_k, b_k geschickt wählt, nämlich

$$a_k := \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(kt)\, dt$$

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Reihe

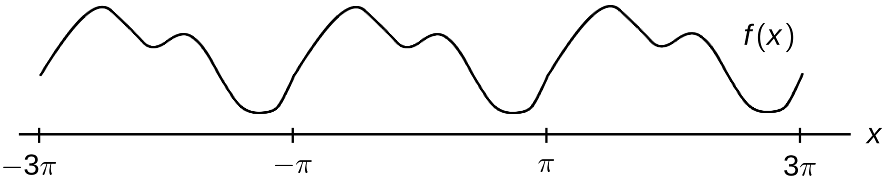
$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx)) = f(x)$$

falls man a_k, b_k geschickt wählt, nämlich

$$a_k := \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(kt)\, dt, \quad b_k := \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(kt)\, dt$$

Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Reihe

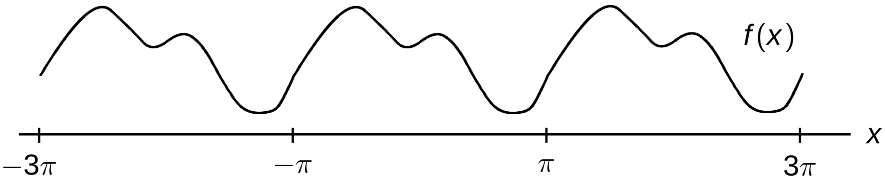
$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx)) = f(x)$$

Fourier-Koeffizienten

$$a_k := \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(kt)\, dt, \quad b_k := \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(kt)\, dt$$

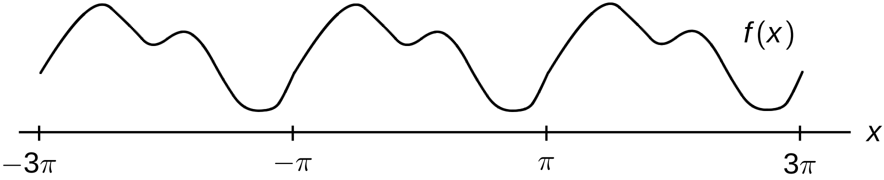
Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Entwicklung

Funktion $f : \mathbb{R} \to \mathbb{R}$, 2π-periodisch, $f(x + 2\pi) = f(x)$



Fourier-Reihe (andere Schreibweise)

Fourier-Entwicklung

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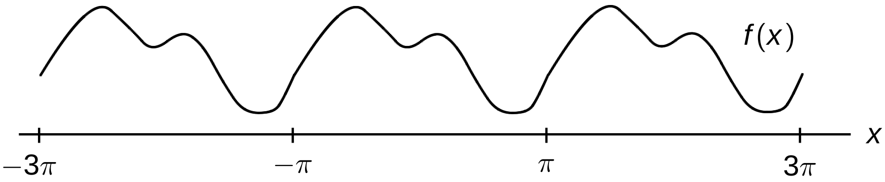

Fourier-Reihe (andere Schreibweise)

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$$

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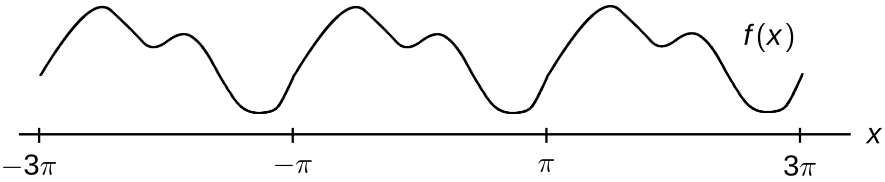

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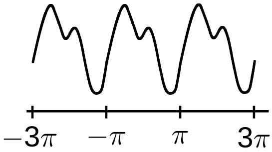
$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$$

Fourier-Koeffizienten

$$c_k := \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-ikt}\, dt$$

Fourier-Entwicklung

f 2π-periodisch



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Fouriertransformation

Fourier-Entwicklung

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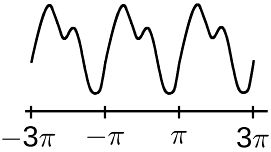

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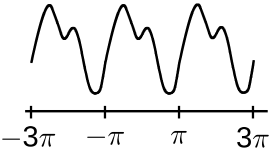

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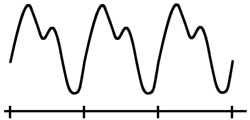
Frage:

Läßt sich die Fourier-Entwicklung auf *nichtperiodische* Funktionen übertragen?

Fourier-Entwicklung

f 2π-periodisch



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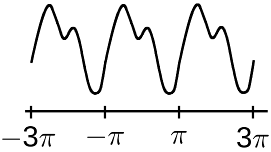

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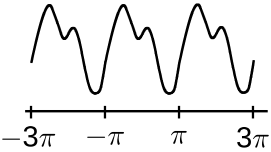

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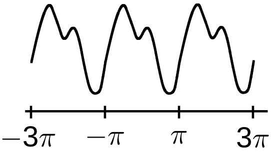

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Fourier-Transformierte

Fourier-Entwicklung

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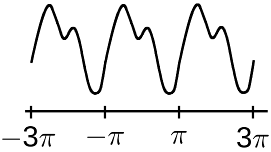

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Fourier-Transformierte

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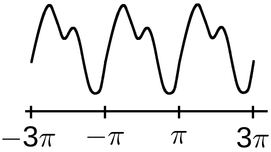
Fourier-Entwicklung

f 2π-periodisch



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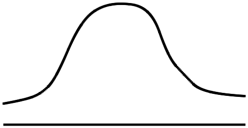
Fouriertransformation

$f : \mathbb{R} \to \mathbb{R}$ nichtperiodisch



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Fourier-Entwicklung

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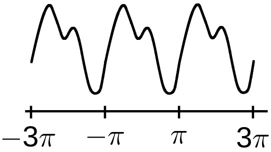

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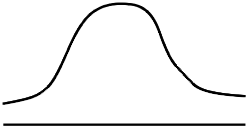


Fourier-Rücktransformation

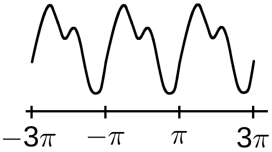
$$f(x) = \int_{-\infty}^{\infty} f^{\wedge}(u) e^{iux}\, du$$

Fourier-Transformierte

$$f^{\wedge}(u) := \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-iut}\, dt$$

Fourier-Entwicklung — Diskrete Fouriertransform.

f 2π-periodisch



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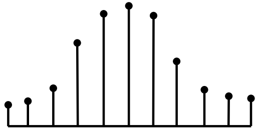
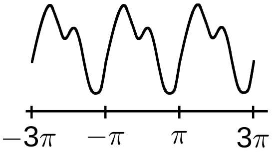
$f_0, \ldots, f_{N-1}$ diskrete Daten



Fourier-Entwicklung

f 2π-periodisch



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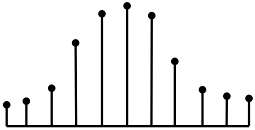


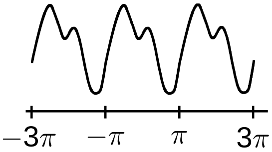
diskrete Fourier-Transformierte

$$F_n := \frac{1}{N} \sum_{k=0}^{N-1} f_k e^{-ikn\frac{2\pi}{N}}$$

$$n = 0, \ldots, N-1$$

Fourier-Entwicklung

f 2π-periodisch



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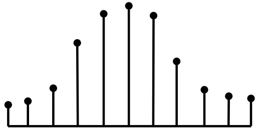

diskrete Rücktransformierte

$$f_n = \sum_{k=0}^{N-1} F_k e^{ikn\frac{2\pi}{N}}$$

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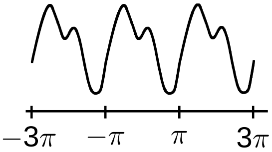
Fourier-Entwicklung

f 2π-periodisch



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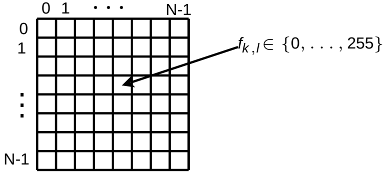
Diskrete 2D-Fouriertransform.

2D-Daten $f_{k,l}$



Fourier-Entwicklung

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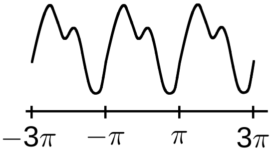

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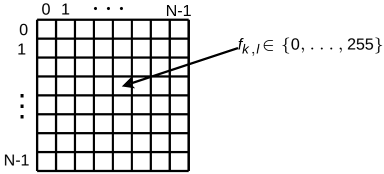


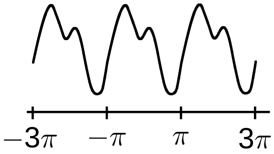
diskrete Fourier-Transformierte

$$F_{n,m} := \frac{1}{N^2} \sum_{k,l=0}^{N-1} f_{k,l} e^{-i(kn+lm)\frac{2\pi}{N}}$$

$$n, m = 0, \ldots, N-1$$

Fourier-Entwicklung

f 2π-periodisch



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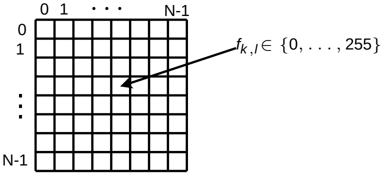

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