Übungen zur Mathematik 1
Lösungen Blatt 11

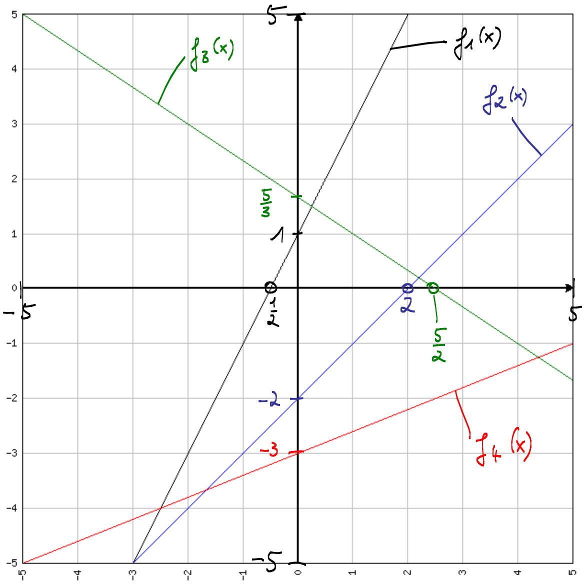
Aufgabe 1

a) $f_1(x) = 2x+1$, Steigung 2, $f(0) = 1$

$f_2(x) = x-2$, " 1, $f(0) = -2$

$f_3(x) = -\frac{2}{3}x + \frac{5}{3}$, " $-\frac{2}{3}$, $f(0) = \frac{5}{3}$

$f_4(x) = \frac{2}{5}x - 3$, " $\frac{2}{5}$, $f(0) = -3$



b) $f_1(x) = 2x+1 = 0 \iff x = -\frac{1}{2}$

$f_2(x) = x-2 = 0 \iff x = 2$

$f_3(x) = -\frac{2}{3}x + \frac{5}{3} = 0 \iff x = \frac{5}{2}$

$f_4(x) = \frac{2}{5}x - 3 = 0 \iff x = \frac{15}{2}$

c) $f_1(x) = f_2(x)$

$\iff 2x+1 = x-2$

$\iff x = -3$

$f_1(-3) = f_2(-3) = -5$ } Schnittpunkt $S_1 = (-3, -5)$

$f_1(x) = f_3(x)$

$\iff 2x+1 = -\frac{2}{3}x + \frac{5}{3}$

$\iff (2+\frac{2}{3})x = \frac{5}{3} - 1$

$\iff \frac{8}{3}x = \frac{2}{3}$

$\iff x = \frac{1}{4}$

$f_1(\frac{1}{4}) = f_3(\frac{1}{4}) = \frac{3}{2}$ } $S_2 = (\frac{1}{4}, \frac{3}{2})$

$f_1(x) = f_4(x)$

$2x+1 = \frac{2}{5}x - 3$

$\iff (2-\frac{2}{5})x = -4$

$\iff \frac{8}{5}x = -4$

$\iff x = -\frac{5}{2}$

$f_1(-\frac{5}{2}) = f_4(-\frac{5}{2}) = -4$ } $S_3 = (-\frac{5}{2}, -4)$

$f_2(x) = f_3(x)$

$\iff x-2 = -\frac{2}{3}x + \frac{5}{3}$

$\iff (1+\frac{2}{3})x = \frac{5}{3} + 2$

$\iff \frac{5}{3}x = \frac{11}{3}$

$\iff x = \frac{11}{5}$

$f_2(\frac{11}{5}) = f_3(\frac{11}{5}) = \frac{1}{5}$ } $S_4 = (\frac{11}{5}, \frac{1}{5})$

$f_2(x) = f_4(x)$

$\iff x-2 = \frac{2}{5}x - 3$

$\iff \frac{3}{5}x = -1$

$\iff x = -\frac{5}{3}$

$f_2(-\frac{5}{3}) = f_4(-\frac{5}{3}) = -\frac{11}{3}$ } $S_5 = (-\frac{5}{3}, -\frac{11}{3})$

$f_3(x) = f_4(x)$

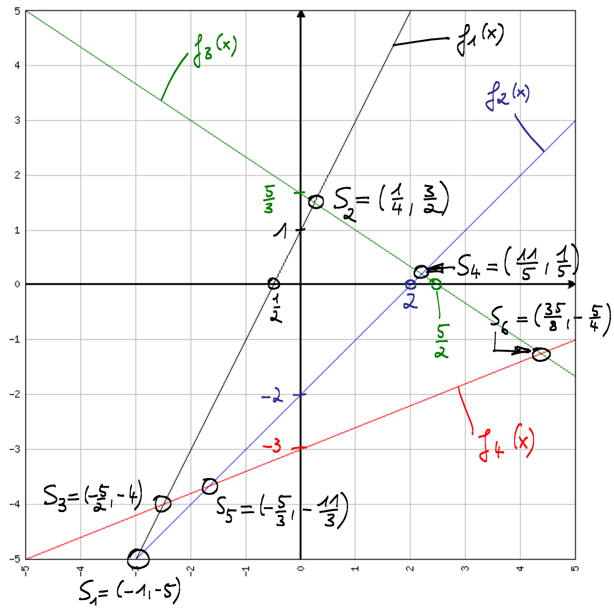
$\iff -\frac{2}{3}x + \frac{5}{3} = \frac{2}{5}x - 3$

$\iff (-\frac{2}{3} - \frac{2}{5})x = -3 - \frac{5}{3}$

$\iff \frac{-16}{15}x = \frac{-14}{3}$

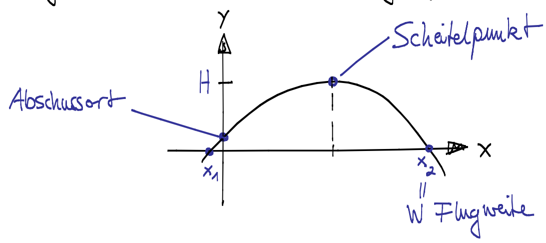
$\iff x = \frac{\cancel{14}^{7}}{\cancel{3}} \cdot \frac{\cancel{15}^{5}}{\cancel{16}_{8}} = \frac{35}{8}$

$f_3(\frac{35}{8}) = f_4(\frac{35}{8}) = -\frac{5}{4}$ } $S_6 = (\frac{35}{8}, -\frac{5}{4})$



Aufgabe 2

Die Flugbahn ist eine nach unten geöffnete Parabel:



Nullstellen: $-\frac{1}{58}(x^2-100x-416)=0$

$\Leftrightarrow \quad x^2-100x-416=0,\ p=-100,\ q=-416$

p,q-Formel: $x_{1,2}=50\pm\sqrt{50^2+416}=50\pm 54$

$x_1=-4\ ,\ \underline{\underline{x_2=104}}$

Flugweite: $x_2=104$ (in Meter)

Scheitelpunkt:

$$f(x)=-\frac{1}{58}(x^2-100x-416)$$
$$=-\frac{1}{58}\left(x^2-100x+\left(\frac{100}{2}\right)^2-\left(\frac{100}{2}\right)^2-416\right)$$
$$=-\frac{1}{58}\left((x-50)^2)-2500-416\right)$$
$$=-\frac{1}{58}(x-50)^2+\frac{2916}{58}$$
$$=-\frac{1}{58}(x-50)^2+50{,}2758...$$

$\Rightarrow$ Scheitelpunkt $S=(50;\ 50{,}2758...)$

Flughöhe: $H\approx 50{,}28$ (in Meter)

Aufgabe 3

a) $2e^{i\frac{\pi}{6}} = 2\cdot\left(\underbrace{\cos\left(\frac{\pi}{6}\right)}_{0{,}866...}+i\,\underbrace{\sin\left(\frac{\pi}{6}\right)}_{0{,}5}\right)$

$= 1{,}732...+i$

b) $3e^{\frac{3}{4}\pi i} = 3e^{i\cdot\frac{3}{4}\pi} = 3\left(\underbrace{\cos\left(\frac{3}{4}\pi\right)}_{-0{,}707...}+i\,\underbrace{\sin\left(\frac{3}{4}\pi\right)}_{0{,}707...}\right)$

$= -2{,}121...+i\cdot 2{,}121...$

c) $5e^{\pi i} = 5\cdot\left(\underbrace{\cos(\pi)}_{-1}+i\cdot\underbrace{\sin(\pi)}_{0}\right)=-5$ reelle Zahl

d) $3e^{\frac{25}{36}2\pi i} = 3\cdot\left(\underbrace{\cos\left(\frac{25}{18}\pi\right)}_{-0{,}342...}+i\,\underbrace{\sin\left(\frac{25}{18}\pi\right)}_{-0{,}939...}\right)$

$= 1{,}026...-i\cdot 2{,}819...$

e) $4e^{i\frac{\pi}{2}} = 4\cdot\left(\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}+i\,\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1}\right)=4i$ imaginäre Zahl

f) $4e^{-i\frac{\pi}{4}} = 4\cdot\left(\underbrace{\cos\left(-\frac{\pi}{4}\right)}_{0{,}707...}+i\,\underbrace{\sin\left(-\frac{\pi}{4}\right)}_{-0{,}707...}\right)$

$= 2{,}828...-i\cdot 2{,}828...$

g) $e^{1024i+5} = e^{1024i}\cdot e^{5}$

$= \left(\underbrace{\cos(1024)}_{0{,}987...}+i\,\underbrace{\sin(1024)}_{-0{,}158...}\right)\cdot\underbrace{e^{5}}_{148{,}4...}$

$= 146{,}5...-i\cdot 23{,}5...$

h) $e^{(1+i)(2-i)} = e^{2-i+2i-(-1)}$

$= e^{3+i} = e^{3}\cdot e^{i}$

$= \underbrace{e^{3}}_{20{,}08...}\cdot\left(\underbrace{\cos(1)}_{0{,}540...}+i\,\underbrace{\sin(1)}_{0{,}841...}\right)$

$= 10{,}85...-i\cdot 16{,}90...$

i) $e^{\frac{1}{1+i}} = e^{\frac{1-i}{(1+i)(1-i)}} = e^{\frac{1-i}{1+1}} = e^{\frac{1}{2}-\frac{1}{2}i}$

$= e^{\frac{1}{2}}\cdot e^{-\frac{1}{2}i} = e^{\frac{1}{2}}\cdot e^{i\left(-\frac{1}{2}\right)}$

$= \underbrace{e^{\frac{1}{2}}}_{1{,}64...}\cdot\left(\underbrace{\cos\left(-\frac{1}{2}\right)}_{0{,}877...}+i\,\underbrace{\sin\left(-\frac{1}{2}\right)}_{0{,}479...}\right)$

$= 1{,}446...-i\cdot 0{,}790...$

j) $e^{(1+i)^4} = e^{(1+2i-1)^2} = e^{(2i)^2} = e^{-4}$

$= 0{,}0183...$ reelle Zahl

Aufgabe 4

a) $\left[2\cdot\left(\cos\left(\frac{\pi}{3}\right)+i\cdot\sin\left(\frac{\pi}{3}\right)\right)\right]^{10}$

$= 2^{10}\cdot\left(e^{i\cdot\frac{\pi}{3}}\right)^{10} = 1024\cdot e^{i\frac{\pi}{3}\cdot 10}$

$= 1024\cdot\left(\underbrace{\cos\left(\frac{\pi}{3}\cdot 10\right)}_{-0{,}5}+i\,\underbrace{\sin\left(\frac{\pi}{3}\cdot 10\right)}_{-0{,}866...}\right)$

$= \underline{\underline{-512-i\cdot 886{,}8...}}$

b) $\left[5\left(\cos\left(-\frac{\pi}{18}\right)+i\cdot\sin\left(-\frac{\pi}{18}\right)\right)\right]^{4}$

$= 5^{4}\cdot e^{-i\frac{\pi}{18}\cdot 4} = 625\cdot\left(\underbrace{\cos\left(-\frac{2\pi}{9}\right)}_{0{,}766...}+i\cdot\underbrace{\sin\left(-\frac{2\pi}{9}\right)}_{-0{,}642...}\right)$

$= \underline{\underline{478{,}77...-i\cdot 401{,}74...}}$