

Übungen zur Mathematik  
Lösungen GSV

Aufgabe 1

$$a) \quad A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 7 \\ 7 \\ 3 \end{pmatrix}.$$

$$D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$b) \quad D^{-1} = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}, \quad L+R = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$-D^{-1}(L+R) = \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \\ 0 & -\frac{1}{2} & 0 \end{pmatrix}$$

$$\begin{array}{l}
 c) \quad 1 + 2 < 4 \quad w \\
 \quad \quad 2 + 1 < 4 \quad w \\
 \quad \quad 0 + 1 < 2 \quad w
 \end{array}$$

A erfüllt das

Zeilensummenkriterium

$\Rightarrow$  GSV konvergiert.

$$\begin{array}{l}
 d) \quad 2 + 0 < 4 \quad w \\
 \quad \quad 1 + 1 < 4 \quad w \\
 \quad \quad 2 + 1 < 2 \quad f
 \end{array}$$

A erfüllt nicht das

Spaltensummenkriterium

e)

Startvektor  $x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Iteration  $x^{(m+1)} = -D^{-1}(L+R)x^{(m)} + D^{-1}b$

$$= \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} x^{(m)} + \begin{pmatrix} \frac{7}{4} \\ \frac{7}{4} \\ \frac{3}{2} \end{pmatrix}, \quad m = 0, 1, 2$$

$$x^{(1)} = \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{7}{4} \\ \frac{7}{4} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{4} \\ \frac{7}{4} \\ \frac{3}{2} \end{pmatrix}$$

$$x^{(2)} = \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} \frac{7}{4} \\ \frac{7}{4} \\ \frac{3}{2} \end{pmatrix} + \begin{pmatrix} \frac{7}{4} \\ \frac{7}{4} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} -\frac{19}{16} \\ -\frac{20}{16} \\ -\frac{14}{16} \end{pmatrix} + \begin{pmatrix} \frac{7}{4} \\ \frac{7}{4} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{9}{16} \\ \frac{21}{16} \\ \frac{5}{8} \end{pmatrix}$$

$$X^{(3)} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{9}{16} \\ \frac{1}{2} \\ \frac{5}{8} \end{pmatrix} + \begin{pmatrix} \frac{7}{4} & \frac{7}{4} \\ \frac{3}{2} & \frac{7}{4} \\ \frac{3}{2} & \frac{7}{4} \end{pmatrix} = \begin{pmatrix} \frac{21}{16} \\ \frac{21}{16} \\ \frac{5}{4} \end{pmatrix}$$

$$X^{(4)} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{21}{16} \\ \frac{21}{16} \\ \frac{5}{4} \end{pmatrix} + \begin{pmatrix} \frac{7}{4} & \frac{7}{4} \\ \frac{3}{2} & \frac{7}{4} \\ \frac{3}{2} & \frac{7}{4} \end{pmatrix} = \begin{pmatrix} \frac{51}{64} \\ \frac{25}{32} \\ \frac{27}{32} \end{pmatrix}$$