

# Übungen zur Mathematik 1

## Lösungen Blatt 6

### Aufgabe 1

$$a) (3a^2 + 5ab + 2b^2) : (a+b) = 3a + 2b$$

$$\begin{array}{r} -(3a^2 + 3ab) \\ \hline 2ab + 2b^2 \\ -(2ab + 2b^2) \\ \hline 0 \end{array}$$

$$b) (3a^2 + 2a - 5) : (3a + 5) = a - 1$$

$$\begin{array}{r} -(3a^2 + 5a) \\ \hline -3a - 5 \\ -(-3a - 5) \\ \hline 0 \end{array}$$

$$c) (a^3 + b^3) : (a+b) = a^2 + b^2 - ab$$

$$\begin{array}{r} -(a^3 + a^2b) \\ \hline b^3 - a^2b \\ -(b^3 + ab^2) \\ \hline -a^2b - ab^2 \\ -(-a^2b - ab^2) \\ \hline 0 \end{array}$$

$$d) (144a^4 - 81b^2) : (36a^2 + 27b) = 4a^2 - 3b$$

$$\begin{array}{r} -(144a^4 + 108a^2b) \\ \hline -81b^2 - 108a^2b \\ -(-81b^2 - 108a^2b) \\ \hline 0 \end{array}$$

0

## Aufgabe 2

$$a) 4^{-2} \cdot 4^4 = 4^{4-2} = 4^2 = 16$$

$$b) \sqrt[3]{-343} = -7$$

$$c) \sqrt[5]{\frac{32}{243}} = \frac{\sqrt[5]{32}}{\sqrt[5]{243}} = \frac{2}{3}$$

$$d) \sqrt[15]{20^{15}} = 20^{\frac{15}{15}} = 20^1 = 20$$

$$e) \sqrt[10]{3^{20}} = 3^{\frac{20}{10}} = 3^2 = 9$$

$$f) \sqrt[20]{2^{30}} = 2^{\frac{30}{20}} = 2^{1,5} = 2^{1+\frac{1}{2}} = 2 \cdot \sqrt{2}$$

$$g) \sqrt[7]{10\,000\,000} = \sqrt[7]{10^7} = 10^{\frac{7}{7}} = 10^1 = 10$$

$$h) \sqrt[3]{0,125} = \sqrt[3]{\frac{125}{1000}} = \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} = \frac{5}{10} = \frac{1}{2}$$

## Aufgabe 3

$$a) \frac{(-1)^{-5}}{(a^{-4})^{-5}} = \frac{-1}{a^{(-4)(-5)}} = \frac{-1}{a^{20}}$$

$$b) (a^{4n^2-1})^{\frac{1}{2n-1}} = a^{\frac{4n^2-1}{2n-1}} = a^{\frac{(2n+1)(2n-1)}{2n-1}} \\ = a^{2n+1}$$

$$c) (x^{\frac{1}{4}})^{\frac{1}{5}} = x^{\frac{1}{4} \cdot \frac{1}{5}} = x^{\frac{1}{20}} = \sqrt[20]{x}$$

$$d) (a \cdot (a \cdot a^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}} = \underbrace{(a \cdot a^{\frac{1}{2}} a^{\frac{1}{4}})^{\frac{1}{2}}}_{a^{\frac{7}{4}}}$$

$$= (a^{\frac{7}{4}})^{\frac{1}{2}} = a^{\frac{7}{4} \cdot \frac{1}{2}} = a^{\frac{7}{8}} = \sqrt[8]{a^7}$$

$$e) 9^{\frac{1}{4}} (3^{\frac{1}{4}})^2 = 3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 3$$

$$f) \underbrace{(x^{\frac{8}{8}} \cdot x^{\frac{3}{8}})}_{x^{\frac{11}{8}}} \cdot x^{\frac{5}{16}} = x^{\frac{11}{16}} \cdot x^{\frac{5}{16}} = x^1 = x$$

### Aufgabe 4

$$a) \sqrt[3]{a^2} - 2 \cdot \sqrt[3]{a} \sqrt[4]{b} + \sqrt{b}$$

$$b) (2\sqrt{a})^2 - (3 \cdot \sqrt[3]{b})^2 = 4a - 9\sqrt[3]{b^2}$$

$$c) a^{2/3} + 2(ab)^{1/3} + b^{2/3} = \sqrt[3]{a^2} + 2\sqrt[3]{ab} + \sqrt[3]{b^2}$$

$$d) \frac{(u-2v)(\sqrt{2u} + \sqrt{4v})}{(\sqrt{2u} - \sqrt{4v})(\sqrt{2u} + \sqrt{4v})} = \frac{(u-2v)(\sqrt{2u} + \sqrt{4v})}{2u - 4v}$$

$$= \frac{\sqrt{2u} + \sqrt{4v}}{2}$$

$$e) \frac{(\sqrt[3]{a^2 x^2} - \sqrt{by})(\sqrt[3]{ax} - \sqrt[4]{by})}{(\sqrt[3]{ax} + \sqrt[4]{by})(\sqrt[3]{ax} - \sqrt[4]{by})}$$

$$= \frac{(\sqrt[3]{a^2 x^2} - \sqrt{by})(\sqrt[3]{ax} - \sqrt[4]{by})}{\sqrt[3]{a^2 x^2} - \sqrt{by}}$$

$$= \sqrt[3]{ax} - \sqrt[4]{by}$$