Übungen zur Mathematik 1
Lösungen Blatt 14

Aufgabe 1

a) $F(x) = \frac{2}{3}x^6 - \frac{3}{2}x^4 + \frac{8}{3}x^3 - \frac{3}{2}x^2 + 5x + C$

b) $F(x) = -3\cos x - 4\sin x + C$

c) $F(x) = 2e^x - 5\ln|x| + x + C$

d) $f(x) = \frac{1}{2x} - x - 2x^2$

$F(x) = \frac{1}{2}\ln|x| - \frac{1}{2}x^2 - \frac{2}{3}x^3 + C$

e) $F(x) = \frac{5}{3}\arctan x - \frac{1}{20}x^5 + C$

f) $F(x) = -2\arcsin x - \tan x + C$

g) $F(x) = -2\cos x - 6\ln|x| + \frac{7}{3}x^3 + C$

h) $F(x) = -3e^x - \sin x + C$

Aufgabe 2

a) $F(x) = e^x - \frac{1}{3}x^3 - x^2 - \cos x + C$

b) $F(x) = e^x + \cot x + C$

c) $\int (4x^2 - 12x + 9)\,dx = \frac{4}{3}x^3 - 6x^2 + 9x + C$

d) $F(x) = -3\arctan x - \ln|x| + C$

e) $F(x) = 5\arcsin x + C$

f) $F(x) = 5e^x - \sqrt{x} + C$

g) $\int \frac{x^{\frac{5}{3}} \cdot x^{\frac{1}{2}}}{x^{\frac{4}{5}}}\,dx = \int x^{\frac{5}{3}+\frac{1}{2}-\frac{4}{5}}\,dx = \int x^{\frac{41}{30}}\,dx$

$= \frac{1}{\frac{41}{30}+1}\,x^{\frac{41}{30}+1} + C = \frac{30}{71}\,x^{\frac{71}{30}} + C$

h) $\int (x\,x^{\frac{1}{2}})^{\frac{1}{2}}\,dx = \int x^{\frac{3}{4}}\,dx = \frac{1}{\frac{3}{4}+1}\,x^{\frac{3}{4}+1} + C$

$= \frac{4}{7}\,x^{\frac{7}{4}} + C$

Aufgabe 3

a) $\int \underset{u'}{x} \cdot \underset{v}{\ln x}\,dx = \underset{u}{\frac{1}{2}x^2} \cdot \underset{v}{\ln x} - \int \overbrace{\underset{u}{\frac{1}{2}x^2} \cdot \underset{v'}{\frac{1}{x}}}^{x}\,dx$

$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{2} \cdot \frac{1}{2}x^2 + C$

$= \frac{1}{2}x^2\left(\ln x - \frac{1}{2}\right) + C$

b) $\int \underset{u}{x} \cdot \underset{v'}{\cos x}\,dx = x\sin x - \int \sin x$

$= x\sin x + \cos x + C$

c) $\int \underset{u'}{1} \cdot \underset{v}{\ln x}\,dx = x\ln x - \int \overbrace{x \cdot \frac{1}{x}}^{1}\,dx$

$= x\ln x - x + C$

$\int_1^5 \ln x\,dx = \left[x\ln x - x\right]_1^5$

$= 5\ln 5 - 5 - (1 \cdot 0 - 1)$

$= 5\ln 5 - 4 = 4{,}047\ldots$

d) $\int \underset{u}{x} \cdot \underset{v'}{\sin(3x)}\,dx = x \cdot \frac{1}{3}(-\cos(3x)) - \int \left(-\frac{1}{3}\cos(3x)\right)dx$

$= -\frac{x}{3}\cos(3x) + \frac{1}{3}\int \cos(3x)\,dx$

$= -\frac{x}{3}\cos(3x) + \frac{1}{9}\sin(3x) + C$

e) $\int \underset{u}{x}\,\underset{v'}{e^x}\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$

$= (x-1)\,e^x + C$

$\int_0^{0{,}8} x e^x\,dx = \left[(x-1)e^x\right]_0^{0{,}8}$

$= -0{,}2\,e^{0{,}8} - (-1 \cdot e^0)$

$= 1 - 0{,}2\,e^{0{,}8} = 0{,}555\ldots$

f) $\int \underset{u'}{1} \cdot \underset{v}{\arctan x}\,dx = x \cdot \arctan x - \int \frac{x}{1+x^2}\,dx$

$= x \cdot \arctan x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

Substitutionsmethode $\Rightarrow x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

$= x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

Aufgabe 4

a) $\int \underset{u'}{e^x}\,\underset{v}{\cos x}\,dx = e^x\cos x + \int \underset{u'}{e^x}\,\underset{v}{\sin x}\,dx$

$= e^x\cos x + e^x\sin x + C - \int e^x\cos x\,dx$

↑ auf die linke Seite bringen

$\Rightarrow 2\int e^x\cos x\,dx = e^x(\cos x + \sin x) + C$

$\Rightarrow \int e^x\cos\,dx = \frac{1}{2}e^x(\cos x + \sin x) + C$

b) $\int \underset{u}{x^2}\,\underset{v'}{e^{-x}}\,dx = -x^2e^{-x} + 2\int \underset{u}{x}\,\underset{v'}{e^{-x}}\,dx$

$= -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,dx\right)$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

$= -e^{-x}(x^2 + 2x + 2) + C$

Aufgabe 5

a) $\int \frac{x^2}{\sqrt{1+x^3}} = \frac{1}{3} \int \frac{\overbrace{3x^2}^{t'}}{\sqrt{1+x^3}}\,dx$

Substitution $t = 1+x^3$

$= \frac{1}{3}\int \frac{1}{\sqrt{t}}\,dt = \frac{1}{3}\int t^{-\frac{1}{2}}\,dt$

$= \frac{1}{3}\,\frac{1}{-\frac{1}{2}+1}\,t^{-\frac{1}{2}+1} + C$

$= \frac{2}{3}\,t^{\frac{1}{2}} + C = 2\sqrt{t} + C$

Rücksubst. $t = 1-x^3$ $\rightarrow = \frac{2}{3}\sqrt{1+x^3} + C$

b) $\int (5x+12)^{\frac{1}{2}}\,dx = \frac{1}{5}\int \underbrace{5}_{t'}\cdot(\underbrace{5x+12}_{t})^{\frac{1}{2}}\,dx$

$= \frac{1}{5}\int t^{\frac{1}{2}}\,dt$

$= \frac{1}{5}\cdot\frac{1}{\frac{3}{2}}\,t^{\frac{3}{2}} + C$

$t = 5x+12 \rightarrow = \frac{2}{15}(5x+12)^{\frac{3}{2}} + C$

c) $\int (1-x)^{\frac{1}{3}}\,dx = -\int(-1)(1-x)^{\frac{1}{3}}\,dx$

Subst. $t = 1-x$

$= -\int t^{\frac{1}{3}}\,dt$

$= -\frac{1}{\frac{4}{3}}\,t^{\frac{4}{3}} + C$

$= -\frac{3}{4}\sqrt[4]{(1-x)^3} + C$

d) $\int \cos^3 x\cdot\sin x\,dx = -\int(\cos x)^3\cdot(-\sin x)\,dx$

Subst. $t = \cos x$

$= -\int t^3\,dt$

$= -\frac{1}{4}t^4 + C$

$= -\frac{1}{4}\cos^4 x$

$\int_0^{\pi} \cos^3 x\cdot\sin\,dx = -\frac{1}{4}\cos^4\pi - (-\frac{1}{4}\cos^4 0)$

$= -\frac{1}{4}(-1)^4 + \frac{1}{4}1^4 = 0$

e) $\int \frac{\arctan x}{1+x^2}\,dx = \int t\,dt = \frac{1}{2}t^2 + C$

Subst. $t = \arctan x$ $\quad = \frac{1}{2}(\arctan x)^2 + C$

f) $\int \frac{2x+6}{x^2+6x-12}\,dx = \int \frac{1}{t}\,dt = \ln|t| + C$

Subst. $t = x^2+6x-12$ $\quad = \ln|x^2+6x-12| + C$

g) $\int \frac{1}{x}\,\frac{1}{\ln x}\,dx = \int \frac{1}{t}\,dt = \ln|t| + C$

Subst. $t = \ln x$ $\quad = \ln|\ln x| + C$

h) $\int x\,\sin(x^2)\,dx = \frac{1}{2}\int 2x\cdot\sin(x^2)\,dx$

Subst. $t = x^2$

$= \frac{1}{2}\int \sin t\,dt$

$= -\frac{1}{2}\cos t + C$

$= -\frac{1}{2}\cos(x^2) + C$

Aufgabe 6

a) $\int_0^4 (x^3-5x^2-1{,}5x-10)\,dx = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 - \frac{3}{4}x^2 - 10x\right]_0^4$

$= 64 - \frac{5}{3}\cdot 64 - 3\cdot 4 - 40 - 0 = -94{,}\overline{6}$

b) $\int_1^e \frac{1}{x}\,dx = [\ln x]_1^e = \ln e - \ln 1 = 1$

c) $\int_0^{\pi}(a\sin x - b\cdot\cos x)\,dx = [-a\cos x - b\sin x]_0^{\pi}$

$= -a\cdot(-1) - (-a) = 2a$

d) $\int_1^4 \frac{1-x^2}{x}\,dx = \int_1^4 (\frac{1}{x} - x)\,dx = [\ln|x| - \frac{1}{2}x^2]_1^4$

$= \ln 4 - 8 - (0 - \frac{1}{2}) = \ln 4 - 7\frac{1}{2} = -6{,}113\ldots$

e) $5\int_1^2 x^{\frac{1}{2}}\,dx = 5\cdot\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^2 = 5\cdot\frac{2}{3}(\sqrt{8} - 1)$

$= 6{,}094\ldots$

f) $\int_{\pi}^{2} \cos x\,dx = [\sin x]_{\pi}^{2} = -\sin 2 - 0 = 0{,}909\ldots$

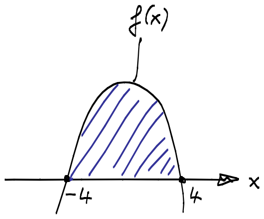
Aufgabe 7

a) $f(x) = -\frac{1}{4}x^2 + 4 = 0$

$\Leftrightarrow \quad \frac{1}{4}x^2 = 4$

$\Leftrightarrow \quad x^2 = 16$

$\Leftrightarrow \quad x = -4 \vee x = 4$

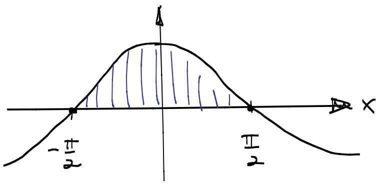


$$\int_{-4}^{4} -\frac{1}{4}x^2 + 4\,dx = \left[-\frac{1}{12}x^3 + 4x\right]_{-4}^{4}$$

$$= -\frac{1}{\cancel{12}\,3}\cdot 4^{\cancel{3}\,2} + 16 - \left(+\frac{1}{\cancel{12}\,3}\,4^{\cancel{3}\,2} - 16\right)$$

$$= 32 - \frac{2}{3}\cdot 16 = 21{,}\overline{3}$$

b) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = \left[\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1 - (-1) = 2$$



c) $f(x) = -3(x-2)^2 + 5$

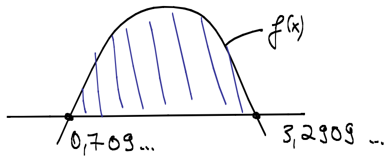
$= -3\,(x^2 - 4x + 4) + 5 = 0$

$\Leftrightarrow \quad x^2 - 4x + 4 - \frac{5}{3} = 0$

$\Leftrightarrow \quad x^2 - 4x + \frac{7}{3} = 0, \quad p = -4, \; q = \frac{7}{3}$

p,q-Formel: $x_1 = 2 - \sqrt{4 - \frac{7}{3}} = 0{,}709\ldots$

$x_2 = 2 + \sqrt{4 - \frac{7}{3}} = 3{,}2909\ldots$



$$\int_{0{,}709\ldots}^{3{,}2909\ldots} \left(-3(x-2)^2 + 5\right) dx$$

$$= \left[-(x-2)^3 + 5x\right]_{0{,}709\ldots}^{3{,}2909\ldots}$$

$$= 8{,}61\ldots$$