KLAUSUR - Vorbereitung : Reelle Zahlen

1.) Man vereinfache:

(a) $8a + (-6b) - 9a - (-13b) + 4a$

(b) $36x - (-12y) + 24y - 14x + (-48y)$

(c) $(7c \cdot (-5c)) + (5d \cdot (-7d))$

2.) Man multipliziere aus:

(a) $(u+v)(u-2v)$ (b) $(2u+3v)(3u-4v)$

(c) $(u+v)(u-2v) \cdot (2u+3v)(3u-4v)$

3.) Wenden Sie die binomischen Formeln an:

(a) $(4a+b)^2$, $(3c-d)^2$, $(8u-5)^2$, $(-g+z)^2$

(b) $(2a-3)(2a+3)$, $(4u-2p)(4u+2p)$

(c) $(a+b)^2-(a^2+b^2)$, $(x+y)^2-(x-y)^2$

4.) Dividieren Sie:

(a) $(6x^3-7x^2-14x+15):(2x+3)$

(b) $(20x^2-23x-20):(5x+3)$

(c) $(x^3-y^3):(x-y)$

5.) Berechnen Sie:

(a) $12 \cdot 3^{-2}$, $a^{-2}x^4ax^{-3}$, $(a^3b^2)^n:(a^2b^3)^n$

(b) $\left(\frac{a}{b}\right)^n\left(\frac{b}{c}\right)^n\left(\frac{c}{a}\right)^{n+1}$, $\left(\frac{x-y}{a+b}\right)^2\left(\frac{a^2-b^2}{x^2-y^2}\right)^2$

(c) $\sqrt[5]{\frac{32}{243}}$, $\sqrt[10]{3^{20}}$, $\sqrt[20]{2^{30}}$, $\sqrt[3]{0{,}125}$

6.) Vereinfachen Sie:

(a) $\sqrt[5]{\sqrt[4]{x}}$ (b) $\sqrt{a\sqrt{a\cdot\sqrt{a}}}$

(c) $\sqrt[4]{9}\cdot\left(\sqrt[4]{3}\right)^2$ (d) $\sqrt{x\cdot\sqrt[8]{x^3}\cdot\sqrt[16]{x^5}}$

7.) Lösen Sie die Klammern auf:

(a) $\left(\sqrt[3]{a} - \sqrt[4]{b}\right)^2$

(b) $\left(2\sqrt{a} - 3\sqrt[3]{b}\right) \cdot \left(2\sqrt{a} + 3\sqrt[3]{b}\right)$

(c) $\left(a^{1/3} + b^{1/3}\right)^2$

Beseitigen Sie die Wurzeln im Nenner durch geeignete Erweiterungen:

(d) $\dfrac{u - 2v}{\sqrt{2u} - \sqrt{4v}}$

(e) $\dfrac{\sqrt[3]{a^2x^2} - \sqrt{by}}{\sqrt[3]{ax} + \sqrt[4]{by}}$

Lösungen

1.) (a) $8a + (-6b) - 9a - (-13b) + 4a$

$= 3a - 6b + 13b = 3a + 7b$

(b) $36x - (-12y) + 24y - 14x + (-48y)$

$= 22x + 12y + 24y - 48y$

$= 22x - 12y$

(c) $(7c \cdot (-5c)) + (5d \cdot (-7d))$

$= -35c^2 - 35d^2 = -35(c^2 + d^2)$

2.) (a) $(u+v)(u-2v) = u^2 - uv - 2v^2$

(b) $(2u+3v)(3u-4v) = 6u^2 + uv - 12v^2$

(c) $(u^2 - uv - 2v^2) \cdot (6u^2 + uv - 12v^2)$

$= 6u^4 + u^3v - 12u^2v^2$

$\quad - 6u^3v - u^2v^2 + 12uv^3$

$\quad - 12u^2v^2 - 2uv^3 + 24v^4$

$= 6u^4 - 5u^3v - 25u^2v^2 + 10uv^3 + 24v^4$

3.) (a) $(4a+b)^2 = 16a^2 + 8ab + b^2$

$(3c-d)^2 = 9c^2 - 6cd + d^2$

$(8u-5)^2 = 64u^2 - 80u + 25$

$(-9+z)^2 = 81 - 18z + z^2$

(b) $(2a-3)(2a+3) = 4a^2 - 9$

$(4u-2p)(4u+2p) = 16u^2 - 4p^2$

(c) $(a+b)^2 - (a^2+b^2) = a^2 + 2ab + b^2 - a^2 - b^2 = 2ab$

$(x+y)^2 - (x-y)^2 = x^2 + 2xy + y^2 - x^2 + 2xy - y^2 = 4xy$

4.) Dividieren Sie:

(a) $(6x^3 - 7x^2 - 14x + 15) : (2x+3) = 3x^2 - 8x + 5$

$$\begin{array}{l}
-(6x^3 + 9x^2) \\
\hline
\quad -16x^2 - 14x \\
\quad -(-16x^2 - 24x) \\
\hline
\qquad 10x + 15 \\
\qquad -(10x + 15) \\
\hline
\qquad\quad 0
\end{array}$$

Probe $(2x+3)(3x^2 - 8x + 5) = 6x^3 - 16x^2 + 10x$
$\qquad + 9x^2 - 24x + 15$
$= 16x^3 - 7x^2 - 14x + 15$

(b) $(20x^2 - 23x - 20) : (5x+3) = 4x - 7$ Rest 1

$$\begin{array}{l}
-(20x^2 + 12x) \\
\hline
\quad -35x - 20 \\
\quad -(-35x - 21) \\
\hline
\qquad\quad 1
\end{array}$$

Probe: $(5x+3)(4x-7) + 1 = 20x^2 - 35x + 12x - 21 + 1$
$= 20x^2 - 23x - 20$

(c) $(x^3 - y^3) : (x-y) = x^2 + xy + y^2$

$$\begin{array}{l}
-(x^3 - x^2y) \\
\hline
\quad x^2y - y^3 \\
\quad -(x^2y - xy^2) \\
\hline
\qquad xy^2 - y^3 \\
\qquad -(xy^2 - y^3) \\
\hline
\qquad\quad 0
\end{array}$$

Probe $(x-y)(x^2 + xy + y^2) = x^3 \cancel{+ x^2y} + \cancel{xy^2}$
$\cancel{- x^2y} - \cancel{xy^2} - y^3$
$= x^3 - y^3$

5.) (a) $12 \cdot 3^{-2} = \frac{12}{3^2} = \frac{4}{3}$

$$a^{-2}x^4 a x^{-3} = a^{-2+1}x^{4-3} = \frac{x}{a}$$

$$(a^3b^2)^n : (a^2b^3)^n = \left(\frac{a^3b^2}{a^2b^3}\right)^n = \left(\frac{a}{b}\right)^n$$

(b) $\left(\frac{a}{\cancel{b}}\right)^n \left(\frac{\cancel{b}}{c}\right)^n \left(\frac{c}{a}\right)^{n+1} = \frac{c}{a}$

$$\left(\frac{x-y}{a+b}\right)^2 \left(\frac{a^2-b^2}{x^2-y^2}\right)^2 = \frac{\cancel{(x-y)^2}}{\cancel{(a+b)^2}} \frac{\cancel{(a+b)^2}(a-b)^2}{(x+y)^2\cancel{(x-y)^2}}$$

$$= \frac{(a-b)^2}{(x+y)^2}$$

c) $\sqrt[5]{\frac{32}{243}} = \frac{\sqrt[5]{32}}{\sqrt[5]{243}} = \frac{2}{3}$

$$\sqrt[10]{3^{20}} = 3^{\frac{20}{10}} = 3^2 = 9$$

$$\sqrt[20]{2^{30}} = 2^{\frac{30}{20}} = 2^{1,5} = 2^{1+\frac{1}{2}} = 2 \cdot \sqrt{2}$$

$$\sqrt[3]{0,125} = \sqrt[3]{\frac{125}{1000}} = \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} = \frac{5}{10} = \frac{1}{2}$$

6.) a) $\left(x^{\frac{1}{4}}\right)^{\frac{1}{5}} = x^{\frac{1}{4}\cdot\frac{1}{5}} = x^{\frac{1}{20}} = \sqrt[20]{x}$

b) $\left(a \cdot \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\underbrace{a \cdot a^{\frac{1}{2}} a^{\frac{1}{4}}}_{a^{\frac{7}{4}}}\right)^{\frac{1}{2}}$

$$= \left(a^{\frac{7}{4}}\right)^{\frac{1}{2}} = a^{\frac{7}{4}\cdot\frac{1}{2}} = a^{\frac{7}{8}} = \sqrt[8]{a^7}$$

c) $9^{\frac{1}{4}}\left(3^{\frac{1}{4}}\right)^2 = 3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 3$

d) $\left(x \cdot x^{\frac{8}{3}}\right)^{\frac{1}{2}} \cdot x^{\frac{5}{16}} = x^{\frac{11}{16}} \cdot x^{\frac{5}{16}} = x^{1} = x$

7.) a) $\sqrt[3]{a^2} - 2 \cdot \sqrt[3]{a}\sqrt[4]{b} + \sqrt{b}$

b) $\left(2\sqrt{a}\right)^2 - \left(3 \cdot \sqrt[3]{b}\right)^2 = 4a - 9\sqrt[3]{b^2}$

c) $a^{2/3} + 2(ab)^{1/3} + b^{2/3} = \sqrt[3]{a^2} + 2\sqrt[3]{ab} + \sqrt[3]{b^2}$

d) $$\frac{(u-2v)(\sqrt{2u}+\sqrt{4v})}{(\sqrt{2u}-\sqrt{4v})(\sqrt{2u}+\sqrt{4v})} = \frac{(u-2v)(\sqrt{2u}+\sqrt{4v})}{2u-4v}$$

$$= \frac{\sqrt{2u}+\sqrt{4v}}{2}$$

e) $$\frac{\left(\sqrt[3]{a^2x^2}-\sqrt{by}\right)\left(\sqrt[3]{ax}-\sqrt[4]{by}\right)}{\left(\sqrt[3]{ax}+\sqrt[4]{by}\right)\left(\sqrt[3]{ax}-\sqrt[4]{by}\right)}$$

$$= \frac{\left(\sqrt[3]{a^2x^2}-\sqrt{by}\right)\left(\sqrt[3]{ax}-\sqrt[4]{by}\right)}{\sqrt[3]{a^2x^2}-\sqrt{by}}$$

$$= \sqrt[3]{ax} - \sqrt[4]{by}$$